An Econometric Identification of Abnormally Low Bids in the Procurement Market: Discriminant Analysis

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Abstract

In the public construction procurement market, 'abnormally low bids (ALB)' are prevalent and they cause many social and economic problems. Also, when the procurement bids are colluded, ALB make the competitive price systematically underestimated. As many countries regulate ALB, their criteria to identify ALB are not homogenous. Most of the criteria are based on construction cost, which is usually inaccurate, vulnerable to accounting manipulation, and limited to the supply side information of the market. We propose an econometric identification process of ALB using a discriminant analysis. It is based on a general mixture model and easily estimable by MLE. We apply our method to Korean public construction bidding data from 2007 to 2016. The estimation results identify the determinants of the bid prices, along with the determinants of ALB, and presents a more accurate assessment of the collusion damage.

JEL Classification: H57, L40, L70

Keywords: abnormally low bids, discriminant analysis, public procurement market

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1. Introduction

The public construction bidding market is unique in two aspects. First, collusions are prevalent in the market. Because the market is usually a monopsony, the competition between suppliers becomes fierce and the winning bid price converges to the marginal cost of the most efficient supplier. Such a severely competitive environment can provide a strong incentive to form a cartel, and thus collusions are often observed. Second, ‘abnormally low bids (ALB)’ are likely to occur in the market. This is because public construction contracts are typically long-term and huge in scope.

The legal definition of ALB is not uniform and somewhat ambiguous. In economic terms, ALB could be defined as ‘a significantly lower bid than the bidder’s marginal cost.’ There are many reasons for ALB: predatory pricing, underestimation of the cost or risk of construction, the intention to change its plan after winning the bid, and etc. We will elaborate these in the next section.

ALB cause many problems. First of all, the competitive price which maximizes the social welfare cannot be achieved when ALB exist in the market. That is the main reason why a lot of countries ban ALB. Another problem of ALB is that they make it difficult to precisely measure the damage of collusion. In order to assess the damage, the competitive price needs to be set as the standard to be compared to a suspected collusion price. When ALB exist in the bidding, however, they erroneously lower the competitive price so that the collusion effect is overestimated.

Nevertheless, it is not an easy task to identify ALB from the bids in public construction procurements. Usually, a ‘normal’ price is calculated from the construction costs, and it is compared to the actual bids to see if any bid is too much lower than the ‘normal’ price. This cost-based identification has a few problems. First, the actual construction costs are rarely observed with accuracy. Second, the cost data are often subject to accounting manipulation. Third, the cost information only reflects the supply side of the market equilibrium, ignoring the demand side information at all.

There have also been efforts to identify ALB through outlier detection analysis. Ballesteros-Pérez et al. (2013), Ballesteros-Pérez et al. (2015), and Skitmore (2002, 2004) among others, have developed bid tendering forecasting models and identified the outliers from the ‘standard pattern’ as ALB (low outliers) and collusion (high outliers). To establish the ‘standard pattern,’ the bidder position distributions, bidding firms’ behaviors are meticulously analyzed. This stylish approach, however, needs the bid data for all participating bidders. As the information on the lost bids is not usually publicized in reality, a practical application of such an outlier detection method could be limited.

We propose in this paper a discriminant analysis to econometrically detect ALB using statistical data on the market. Discriminant analysis is first formulated by Fisher (1936), and developed by Goldfeld and Quandt (1972), Kiefer (1980), Quandt and Ramsey (1978) and Schmidt (1982). In the field of industrial economics, Spiller and Huang (1986) apply a discriminant analysis utilizing a stochastic frontier switching regression to regional market definition problem. A more visible example of discriminant analysis is collusion detection problem. Porter (1983), Lee and Porter (1984), and Ellison (1994) use a discriminant analysis, more specifically, 'switching regression model with imperfect sample separation information,' to detect the collusion periods from weekly time series data on the Joint Executive Committee (JEC) railroad cartel from 1880 to 1886. Their method discriminates collusion from competitive contracts utilizing an imperfectly observed switching (between collusion and competition) information. As the railroad cartel data they use include error-ridden partial information on collusion, the choice of their method is natural.

In this paper, we propose a general mixture regression model based on Kiefer (1980) to discriminate ALB from normal contracts without utilizing any prior switching information. Our discrimination method can be applied to any regime switching model including the collusion detection models cited above when no prior information is available about the switching status. We apply our method to Korean public construction bidding data from 2007 to 2016, and empirically identify ALB.

2. Abnormally Low Bids

The definition of ALB varies. Albano (2017) surveys the various standards of ALB in the international law. The 'UNCITRAL Model Law on Public Procurement 2011' by the United Nations states: "The procuring entity may reject a submission if the procuring entity has determined that the price, in combination with other constituent elements of the submission, is abnormally low in relation to the subject matter of the procurement and raises concerns with the procuring entity as to the ability of the supplier or contractor that presented that submission to perform the procurement contract." The World Trade Organization states: "Only tenders that conform to the essential requirements of the tender notice or documentation and are from a supplier which complies with the conditions for participation can be considered for award. Entities have the obligation to award contracts to the tenderer who has been determined to be fully capable of undertaking the contract and whose tender is either the lowest tender or the tender which is determined to be the most advantageous in terms of the specific evaluation criteria set forth in the notices or tender documentation. An entity that has received a tender abnormally lower than other tenders may enquire with the tenderer to ensure that it can comply with the conditions of participation and be capable of fulfilling the terms of the contract." (Article XIII: 4).

According to the Federal Acquisition Regulation of the U.S. government, ALB can be referred to as unfair and unreasonable bids compared to the preliminary estimate of a client.[4] The European Union explicitly obliges its member states to explain the price or costs contained in a tender in situations where tenders "appear to be abnormally low in relation to the works, goods or services". The European Union provides the following guidelines as to the elements of a tender price may be subjected to further scrutiny:[5]

- in the light of client's preliminary estimate & of all the tenders submitted, it seems to be abnormally low by not providing a margin for a normal level of profit
- In relation to which the tenderer cannot explain his price on the basis of the economy of the construction method, or the technical solution chosen, or the exceptionally favorable conditions available to the tenderer, or the originality of the work proposed.

In sum, ALB can be legally defined as the bid whose price is unreasonably low so that the bidder cannot profit from the contract. In economic terms, ALB could be defined as 'a significantly lower bid than the bidder's marginal cost.'

The reasons why ALB occurs in construction procurements are manifold. OECD (2016) and Ibrahimi (2017) list the following reasons. First, the bidder misunderstands the conditions or details of construction contract. Second, the bidder underestimates the risk of the contract and submits a very low price. Third, the bidder pursues illegal profits by not complying with the essential laws involved with the construction. Fourth, a government subsidy can make a

[4]Carpineti et al. (2006). Although the U.S. Federal Acquisition Regulation does not define ALB explicitly, it emphasizes that all prices during the procurement procedure should be fair and reasonable. The footnote 27 in Carpineti et al. (2006) explains shat 'fair and reasonable prices' are.

[5] See Harrower (1999) for a detailed explanation.

bid look like an ALB as the bidder offers a lower price than the other bidders without subsidy.

Sometimes, bidders use ALB as a strategy. First, a company would take the risk of a significant loss if it is desperate in winning the contract due to cash flow problem. As public constructions contracts are typically long-term, some companies may take the short-term loss to stabilize the income stream. Gunduz and Karacan (2008) survey on the causes of ALB in Turkish government procurements. They find that the hope of staying in the business is the most important reason. Calveras et al. (2004) also argue that a firm in financial trouble bids more aggressively with a lower price in order to survive in the market.

Second, firms utilize ALB as a predatory pricing strategy. In other words, they submit an abnormally low bid to drive competitors out of the market. Alexandersson and Hulten (2006) find ALB as predatory pricing by analyzing Swedish train service data in 2002. Bedford (2009) also argues that firms' predatory intentions cause ALB and that a prequalification procedure can reduce predatory ALB. Third, when the bidder considers re-negotiations after winning the contract, ALB may occur. Calveras et al. (2004) claim that one of the main reasons for ALB is the expectation for re-negotiation at the time the client cannot change the contractor.

ALB makes a number of problems. First, the contractor with ALB faces many risks: a default risk, a risk of paying additional costs during implementing the contract, a risk of not abiding by the laws or the contract terms.[6] Second, quality deterioration is also possible due to ALB. If the quality of the construction becomes worse, the consumers' satisfaction and social welfare will be lowered. Third, ALB can remove the competitors out of the market by predatory pricing. Such reduction in competition would be harmful to consumers.

In addition to these negative effects, ALB makes it difficult to assess the damage of collusion in the market. The damage of collusion is usually measured by the difference between the competitive price and the actual price. As the price of ALB is most likely a low-end outlier, the existence of ALB in the market tends to bias the estimated competitive price downward. Accordingly, the collusion damage would most likely to be overestimated.

For these reasons, many countries regulate ALB. Of course, the major problem in the regulation is the difficulty in identifying ALB. All the current regulations are based on the observed bid price. The criterion to determine a bid 'too low,' however, varies country by country. For example, Belgium determines a bid to be an ALB if the bid is lower than 85% of the mean bid. Portugal compares the bids to the estimated base price. If the bid is lower than

[6] OECD(2016), Ibrahimi(2017)

60% of the base price, the bid is determined as an ALB.

In general, the criteria for ALB could be categorized by 'absolute criterion' and 'relative criterion.' An absolute criterion evaluates the deviation of a bid price from the client's preliminary estimate of the price. If the bid is lower than the allowed deviation, it is determined 'abnormally low.' Though the absolute criterion can be applied regardless of the number of bidders, it depends on the accuracy of the estimated price. A relative criterion uses the deviation from the mean of the other bid prices. To avoid a distortion, most countries preset the minimum number of bidders, or exclude the maximum and minimum bids. While the relative criterion does not need any pre-determined price estimate, it is difficult to apply when the number of bidders is small. Some countries combine both the absolute and relative criteria. Table 1 summarizes the criteria of ALB for selected countries.

<Table 1 > ALB Criteria in Selected Countries

Country	Type of Criteria	Applied Cases	Criteria	Notes
World Bank	Absolute	bidders<5	20% or more below the Borrower's cost estimate	World Bank (2016)
	Relative	bidders≥5	More than one standard deviation below the average of the substantially responsive bids received	
Belgium	Relative	bidders≥4	15% lower than the mean of the bids if at least 4 bids are submitted (the mean refers to the mean of all the bids apart from the highest and the lowest if the bids are equal or more than 7)	Megremis (2013)
Bulgaria	Relative	-	20% lower than the mean of the other bids (30% until 2012)	Public Procurement Act, Article 70.
Hungary	Absolute	-	20% lower than the available funds (more flexibility introduced after 2015)	Act CXLIII of 2015 on Public Procurement, Article 69.
Italy		-	Points scored in price and quality are both more than 4/5 of the corresponding maximum points (which implies low price for high quality)	OECD (2016)
Portugal	Absolute	-	40% lower than the base price in the specifications	Megremis (2013)

Romania	Absolute	bidders<5	Less than 85% of the estimated value of the contract	Until 2016
	Relative	bidders≥5	Less than 85% of the arithmetic average of the price of the submitted tenders, without taking into account the lowest and highest prices proposed	
Slovenia	Relative	bidders≥4	50% lower than the mean of the bids and 20% lower than the 2nd lowest bid if at least 4 bids are submitted	The Public Procurement Act (ZJN-3), Official Gazette no. 91/15, Article 86.
Spain	Absolute	1 bidder	25% lower than the base price of the contract	Megremis (2013)
	Relative	2 bidders	20% lower than the second bid if 2 bids received	
	Relative	3 bidders	10% lower than the mean of all the bids, but if the highest bid is 10% higher than the mean of all bids it should be excluded from the calculation of the mean	
	Relative	bidders≥4	10% lower than the mean of all the bids, but all bids that are higher than 10% from the mean of all bids should be excluded from the calculation of the mean and if the remaining bids are less than 4 then one of the above three rules should be applied.	
Poland	Absolute Relative	-	Lower by at least 30% from the contract value or the arithmetic mean of the prices of all tenders submitted	ACT of 29 January 2004 PUBLIC PROCUREMENT LAW
Slovakia	Relative	bidders≥3	At least 15% lower than the average price of all other offers or at least 10% lower than the second lowest offer.	OECD (2016)
Turkey		-	Uses a preset formula based on the arithmetic mean of the bids (by excluding bids which are over %120 of conceptual cost and below %40 of conceptual cost)	Public Procurement Law, Article 38 / Karacan(2008)
Brazil	Absolute Relative	-	70% lower than the lowest of the following values: (1) the arithmetic average between tendering prices that are superior than 50% of the	Carpineti et al. (2006)

			estimated price set by the Administration; (2) the estimated price set by the Administration.	
U.S.A.	Absolute	-	Lower than 75% or higher than 150% of the government estimate (Wisconsin) Lower or higher by more than 15% of the government estimate (New York)	Choi (2010), Downing (2004)
Japan	Absolute	-	Lower than 70%~85% of the estimated price (different by provinces)	Choi (2010, 2011)

Conti and Naldi (2008) and Ballesteros-Pérez et al. (2015), among others, mathematically simulate the accuracy and efficiency of the above screening strategies for ALB. They show that many factors such as the number of bids and the dispersion of bids affect the performance of the criteria, and that the criteria could often be significantly misleading. Fuentes-Bargues et al. (2016) argues that the above ALB criteria using the price of bids would be risky and other factors, such as labor involved in the contract, economic improvements and the guarantee period need to be considered along with the price.

By all means, it is not easy to identify ALB from the bids in public construction procurements. Needless to mention, to construct a 'normal' price from the construction costs has more problems than any reality. Besides the actual construction costs are rarely observed, the cost data are often subject to accounting manipulation. Moreover, the cost information only reflects the supply side of the market equilibrium, ignoring the demand side information at all.

As an alternative approach, there have been efforts to identify ALB through outlier detection analysis. Ballesteros-Pérez et al. (2013), Ballesteros-Pérez et al. (2015), and Skitmore (2002, 2004) among others, have developed bid tendering forecasting models and identified the outliers from the 'standard pattern' as ALB (low outliers) and collusion (high outliers). To establish the 'standard pattern,' the bidder position distributions, bidding firms' behaviors are meticulously analyzed. This approach, however, needs the bid data for all participating bidders, including the lost bids. As the information on the lost bids is not usually publicized in reality, a practical application of such an outlier detection method could be limited.

3. Discriminant Analysis

This paper proposes a discriminant analysis for econometrical identification of ALB from procurement data. Discriminant analysis is a statistical method separating a distribution from a mixture of distributions. Fisher (1936) first shapes up the method, and a number of procedures have been developed by Goldfeld and Quandt (1972), Kiefer (1980), Quandt and Ramsey (1978) and Schmidt (1982), among others. Our method utilizes a mixture regression model similar to Kiefer (1980) to separate ALB from the mixture distribution of ALB and normal bids.

We consider the following price equation in reduced form.

$$P_t = \beta' X_t + \delta I_t + u_t \tag{1}$$

where P_t is the price (or the price ratio to the preliminary estimate) of a winning bid, X_t is a set of variables explaining the winning price of bid *t*, for example, a dummy variable for collusion (1 if there is a collusive behavior in bid *t*, 0 otherwise), the number of bidders, the business cycle index, bidding types or period effects, etc. I_t is a dummy variable for ALB; 1 if it is ALB, 0 otherwise. u_t is an error term of the regression model. As the ALB dummy will lower the price, the coefficient δ is expected to have a negative value.

In reality, ALB dummy, I_t, is not observed a priori. We assume that the unobservable I_t has the following binomial distribution:

$$I_t = 1 \quad \text{with probability} \quad \lambda_t \tag{2}$$

$$I_t = 0 \quad \text{with probability} \quad 1 - \lambda_t \tag{3}$$

In (2), λ_t is the unconditional probability that bid *t* is actually an ALB. In discriminant analyses, this unconditional probability is often assumed to be a constant, as in Lee and Porter (1984). It is unrealistic, however, to assume all of the bids should have the same probability of $I_t = 1$. For example, if there are more bidders in the bidding, ALB are more likely to occur because of the competitiveness. Types of bidding or the scope of work might also change the possibility of ALB. To incorporate the possibility of heterogeneous unconditional probability of ALB, we specify λ_t as a function of multiple covariates. As λ_t is a probability function, we employ a logistic function similar to Ellison (1994).[7]

$$\lambda_t = \frac{e^{\alpha' Z_t}}{1+e^{\alpha' Z_t}} \tag{4}$$

where Z_t is a set of variables which affect the occurrence of ALB.

From equation (1) through (4), we can derive the likelihood function of the data generation process. The likelihood function of the regression model is as follows:

$$L = \Pi\left[f_1(P_t)\lambda_t + f_2(P_t)(1-\lambda_t)\right]$$

(5)

where $f_1(P_t)$ is the probability density function of bid price if the bid is actually an ALB (i.e. $I_t = 1$), while $f_2(P_t)$ is the probability density function if the bid is not an ALB (i.e. $I_t = 0$). We assume that the bid price follows a normal distribution as follows:

$$f_1(P_t) = \frac{1}{\sqrt{2\pi}\sigma}\exp\{-\frac{1}{2\sigma^2}(P_t - \beta' X_t - \delta)^2\} \tag{6}$$

$$f_2(P_t) = \frac{1}{\sqrt{2\pi}\sigma}\exp\{-\frac{1}{2\sigma^2}(P_t - \beta' X_t)^2\} \tag{7}$$

After substituting (6) and (7) into (5), the parameters in the regression model, β, α and σ^2 are estimated by maximizing the likelihood function (5).[8] With the estimates of the parameters, we can calculate the estimated conditional probabilities, $\Pr(I_t = 1 | P_t)$ and $\Pr(I_t = 0 | P_t)$ for each observation. By comparing the two conditional probabilities, we can decide which distribution the observation belongs to. Lee and Porter (1984) show that the simplest rule is the best: if $\Pr(I_t = 1 | P_t) > \Pr(I_t = 0 | P_t)$ then the bid is discriminated as an ALB, and if $\Pr(I_t = 1 | P_t) < \Pr(I_t = 0 | P_t)$ then the bid is discriminated as a normal bid.[9] As the sum of the two conditional probabilities must be 1, the rule can be also stated as: the bid is discriminated as an ALB if $\Pr(I_t = 1 | P_t) > 0.5$.

[7] As Ellison (1994) deals with time-series data, he uses a Markov structure for the logit function. We employ a contemporary logit structure, as we apply our method to cross-section data.

[8] We use GAUSS and R for the numerical maximization.

[9] Lee and Porter(1984), pp 400-401.

4. Empirical Analysis

In this section, we apply the above discriminant analysis to actual procurement data. The empirical data are on Korean public construction bidding from 2007 to 2016.[10] They are collected from the Public Procurement Service (PPS) of Korea and the Construction Association of Korea (CAK). Among the 864 procurement contracts in 2007-2016, 111 contracts (about 12.8%) has been detected as colluded by KFTC (Korean Fair Trade Commission).[11] The following variables are collected for the 864 contracts.

(1) Bid Price Ratio: the ratio of winning bid price to the contractor's preliminary estimate
(2) Collusion dummy: a collusion indicator; 1 if the bid has been detected as colluded by KFTC investigation, 0 otherwise.[12]
(3) Number of Bidders: total number of the bidders in the bid
(4) CBSI: Construction Business Survey Index by the Bank of Korea
(5) Type of Construction: dummy variables for five types, 'architecture,' 'plant,' 'civil engineering,' 'landscaping' and 'railroad'.
(6) Type of Bidding: dummy variables for four types, 'lowest price,' 'turn-key,' 'alternative,' and 'technical proposal.' For lowest price bidding, the bidders present only the price bid, while the bidders present price and construction plan for turn-key bidding. For alternative bidding, the bidders present an alternative construction plan and price along with the price for the original plan proposed by the procuring agent. For technical proposal, the bidders present an alternative plan, alternative technology and price.
(7) Year Dummies: 9 year dummies are used as the data are collected for 2007-2016. The year of 2016 is the base year.
(8) ALB: an abnormally low bid indicator; 1 if it is ALB, 0 otherwise.

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[10] For the structure and regulations in Korean public construction procurement market, see Jeong and Lee (2018).

[11] We use only the bids in which the number of bidders are less than 21. The reasons are: first, any bid with more than 20 bidders is regulated by a different Pre-Qualification standard, second, those bids with more than 20 bidders are usually very small construction project.

[12] Thus, there exists a possibility of under-detection for the collusion dummy.

The descriptive statistics of the above variables are listed in <Table 2>. Bid price ratio ranges from 41.51% to 100%.[13] That the lowest bid price ratio is as low as 41.51% implies a possibility of abnormally low tender. The number of bidders is about 4.3 on average but left-skewed: only two bidders compete in more than half the contracts. CBSI, an indicator for business cycle in the construction industry, varies from 65 to 105 and its average is 82.64. Types of construction in the data set consists of five categories: architecture (30.9%), railroad (9.49%), civil engineering (40.86%), landscaping (0.69%), plant (base category, 18.06%). There are four types of bidding: turn-key (67.36%), alternative (6.94%), technical proposal (9.95%), lowest price (base category, 15.74%). The data cover 10 year period of 2007-2016. One interesting fact is more than a quarter of the whole 864 contracts are in the year of 2009 (243 contracts, 28.13%). The reason is the 'Four River Refurbishment Project' by then-President Lee Myung-bak. Massive number of contracts from the total budget of 22.2 trillion won (about US$20 billions) were made in 2009.

<Table 2> Descriptive Statistics

	Count	Mean	Std Dev	Min	25%	Median	75%	Max
Bid Price Ratio	864	0.8878	0.1212	0.4151	0.8209	0.9449	0.9744	1.0000
Collusion Dummy	864	0.1284	0.3348	0	0	0	0	1
Number of Bidders	864	4.2708	4.4449	2	2	2	4	20
CBSI	864	82.64	9.7208	65	75	81	89	105
Architecture	864	0.3090	0.4624	0	0	0	1	1
Railroad	864	0.0949	0.2933	0	0	0	0	1
Civil Engineering	864	0.4086	0.4919	0	0	0	1	1
Landscaping	864	0.0069	0.0831	0	0	0	0	1
Turn-key	864	0.6736	0.4692	0	0	1	1	1
Alternative	864	0.0694	0.2544	0	0	0	0	1
Technical Proposal	864	0.0995	0.2996	0	0	0	0	1
Lowest Price	864	0.1574	0.1328	0	0	0	0	1

[13] The maximum bid rate is 1 as the bid was exactly the same as the preliminary estimate in a contract (Juam Dam water conveyance tunnel stabilization project).

Year 2007	864	0.0174	0.1307	0	0	0	0	1
Year 2008	864	0.1157	0.3201	0	0	0	0	1
Year 2009	864	0.2813	0.4499	0	0	0	1	1
Year 2010	864	0.1250	0.3309	0	0	0	0	1
Year 2011	864	0.1331	0.3399	0	0	0	0	1
Year 2012	864	0.0822	0.2748	0	0	0	0	1
Year 2013	864	0.0926	0.2900	0	0	0	0	1
Year 2014	864	0.0498	0.2176	0	0	0	0	1
Year 2015	864	0.0613	0.2401	0	0	0	0	1
Year 2016	864	0.0417	0.1999	0	0	0	0	1

The regression model we estimate is as follows.

$$P_t = \beta' X_t + \delta I_t + u_t \quad (10)$$

$$\lambda_t = \frac{e^{\alpha' Z_t}}{1 + e^{\alpha' Z_t}} \quad (11)$$

where P_t is the bid price ratio, X_t is a vector of explanatory variables explaining the bid price: a constant, a collusion dummy, the number of bidders, CBSI, types of a construction, types of a bidding, and year dummies. CBSI and year dummies are included to capture the variation in the demand side of the construction business. I_t is the ALB indicator. As mentioned in section 3, I_t is unobservable. We specify the probability of I_t being 1 as in equation (11). Z_t includes a constant, the logarithm of the number of bidders, CBSI and types of a construction.

The maximum likelihood estimation result of equation (10) is presented in <Table 3>.

<Table 3> Maximum Likelihood Estimation Result of Equation (10)

Variable	Estimates	Standard Error	t-value
Constant	0.8744	0.0243	36.0126***
Collusion Dummy	0.0109	0.0059	1.8574**
Log(number of bidders)	-0.0402	0.0058	-6.9661***

CBSI	0.0000	0.0002	0.0306
Architecture	0.0220	0.0063	3.4741***
Railroad	0.0117	0.0080	1.4490*
Civil Engineering	-0.0160	0.0057	-2.8221***
Landscaping	-0.0090	0.0213	-0.4205
Turn-key	0.1354	0.0113	12.0306***
Alternative	0.0872	0.0125	6.9933***
Technical Proposal	0.1064	0.0145	7.3271***
Year 2007	-0.0259	0.0189	-1.3719*
Year 2008	-0.0297	0.0118	-2.5292***
Year 2009	-0.0279	0.0112	-2.4930***
Year 2010	-0.0294	0.0117	-2.5119***
Year 2011	-0.0209	0.0115	-1.8256**
Year 2012	-0.0281	0.0117	-2.4011***
Year 2013	-0.0260	0.0123	-2.1199**
Year 2014	-0.0276	0.0127	-2.1735**
Year 2015	-0.0112	0.0138	-0.8142
ALB	-0.2822	0.0071	-39.5281***

(*: 10% significance level, **: 5% significance level, ***: 1% significance level)

First of all, the collusion effect estimate is 1.09% and is statistically significant at 5% level. This means that the collusive behavior makes the bid price ratio 1.09%p higher than the competitive price. This is a somewhat lower collusion effect than in other industries. Connor (2007) surveys 674 long-run cartels and finds that the median collusion overcharge is 25%. Our estimate, 1.09%, is certainly quite lower than such median. The reason might be because the client of public construction procurement is usually Korean government. Korean government tends to set the preliminary cost estimate (which is the maximum bid price) of public construction very conservatively. As the cost estimate is already very low, it is difficult for a competitive bid to be much lower than that. Thus even if the bids are colluded, there is not enough room to increase the bid highly over the competitive price. Of course, if we would not consider ALB in the estimation of the price equation, the collusion damage is estimated significantly higher than 1.09%. Though we suppress the detailed estimation result to conserve space, the collusion effect is estimated as 5.16% (almost five times higher than 1.09%) if the effect of ALB is not controlled. It is confirmed that ALB makes the collusion damage significantly overestimated.

Second, as the number of bidders increases by 1%, the price bid ratio falls by 4% and it is statistically significant at 1% level. It is rational to say that the competition gets fiercer when there are more bidders. Third, the coefficient of Construction Business Survey Index (CBSI) is not statistically significant. Fourth, for construction types, compared to plant construction, the bid prices of architecture construction and railroad construction are 2.20%p and 1.17%p higher respectively and statistically significant at the 1% level and 10% level respectively. Civil construction significantly lowers the bid price ratio by 1.60%p, and landscaping does not affect the price ratio.

Fifth, it is intuitive that all the bidding types, turn-key, alternative, and technical proposal, produce higher price ratio than the lowest price bidding. Sixth, the year dummies compared to the base year of 2016 show the upward trend of bid price ratio. Last, when all the other variables are held constant, the ALB contracts are estimated to lower the average bid price ratio by 28.22%p.

<Table 4> presents the estimation result of equation (11), the logistic probability of ALB. It is shown that the higher the number of bidders, the higher the probability of ALB. This is plausible in the sense that a company would be inclined to ALB if the competition is severe. Second, the result shows that the probability of ALB becomes lower, if the construction industry is in prosperous business cycle. Third, among the various types of construction, architecture and railroad constructions significantly lower the probability of ALB. That a successful architecture construction requires more creativity and technology than price advantage explains the estimated coefficient.

<Table 4> Maximum Likelihood Estimation Result of Logistic Probability (11)

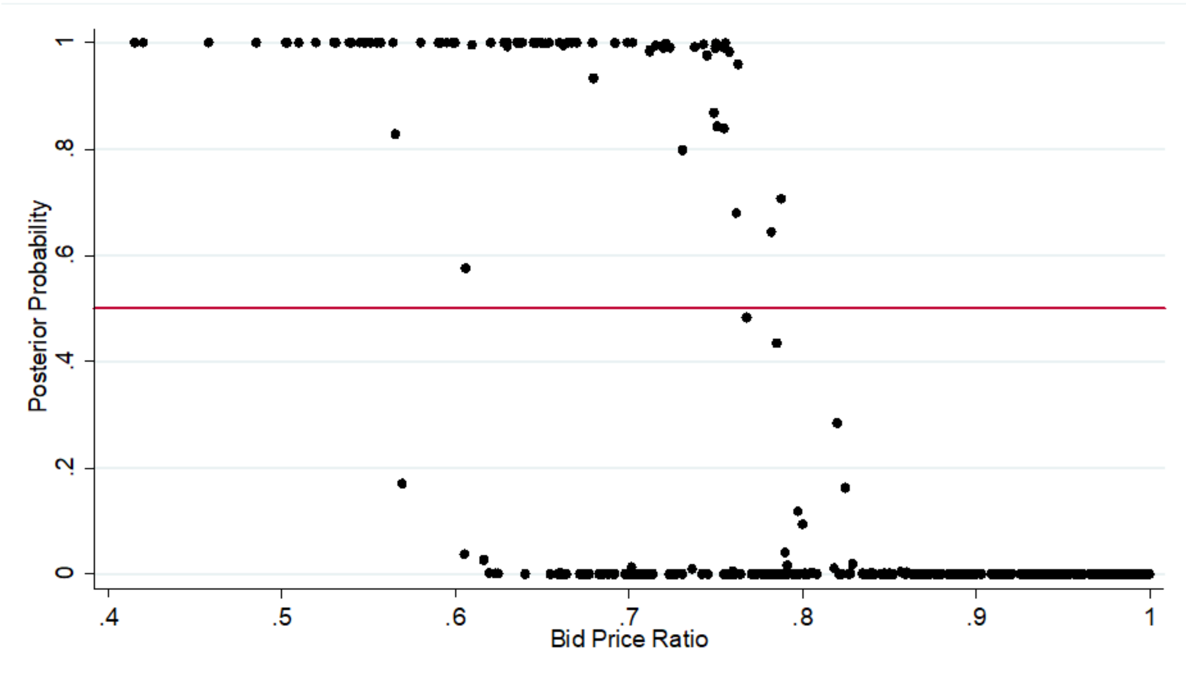
Variable	Estimates	Standard Error	t-value
Constant	-1.1942	1.1419	-1.0458
Log(number of bidders)	0.0714	0.1951	0.3660
CBSI	-0.0084	0.0129	-0.6534
Architecture	-1.0613	0.3776	-2.8105***

Railroad	-1.9496	0.7831	-2.4896***
Civil Engineering	-0.1794	0.2973	-0.6034
Landscaping	0.2168	1.1230	0.1931

(*: 10% significance level, **: 5% significance level, ***: 1% significance level)

As a result from the maximum likelihood estimation of the discriminant analysis model, 82 contracts out of 864 observations (about 9.5%) are classified as ALB. <Figure 1> presents the classification results. The vertical axis of Figure 1 measures the posterior probability of ALB, i.e. $\Pr(I_t = 1 | P_t)$, and the horizontal axis measures the bid-price ratio. The contracts above the red horizontal line (at posterior probability of 0.5) have been categorized as ALBs by our discriminant analysis.

<Figure 1> Graph of Conditional Probability



<Table 5> summarizes the difference between these two groups: ALB and NB (normal bids). First, the average bid price ratio of ALBs (0.6410) is about two-third of the average bid price ratio of NBs (0.9137). Second, no ALB is identified out of the 111 colluded contracts (14.19% out of 782 normal bids). Naturally, no firm would have any incentive for ALB when the contract is colluded. Third, the average number of bidders of NBs (4.3555) is a bit higher

than the average number of bidders of ALBs (3.4634). Considering the high standard deviations (4.6274 and 1.8203), however, the difference may not be statistically significant. Fourth, business cycle does not seem to affect the decision on ALB. The average CBSI are almost identical regardless the bid is ALB or NB. Fifth, the type of construction seems to be related with ALM decision. More than half (52.44%) of the ALBs in the data are civil engineering construction. That the 'quality' of construction is relatively harder to verify in civil engineering probably attributes its high percentage in ALBs. Sixth, the type of bidding also seems to affect ALB decision. ALB occurs more in turn-key type bidding and does less in lowest price type bidding. While only 66.24% (518 contracts) out of 782 NBs are turn-key type, 78.05% (64 contracts) out of 82 ALBs are turn-key type. Contrary to turn-key bidding, 17.14% of NBs are lowest price bidding while only 2.44% of ALBs are lowest price bidding. A possible explanation would be that the bidder has more flexibility in turn-key type construction. As only a price is tendered in lowest price type bidding, a price along with a construction plan are presented in turn-key type bidding. Seventh, there is no distinct yearly difference between NBs and ALBs.

<Table 5>

	NB (Normal Bids)				ALB (Abnormally Low Bids)			
	Mean	Std Dev	Min	Max	Mean	Std Dev	Min	Max
Bid Price Ratio	0.9137	0.0914	0.5695	1	0.6410	0.0894	0.4151	0.7878
Collusion Dummy	0.1419	0.3492	0	1	0	0	0	0
Number of Bidders	4.3555	4.6274	2	20	3.4634	1.8203	2	12
CBSI	82.6752	9.8295	65	105	82.3049	8.6634	65	105
Architecture	0.3235	0.4681	0	1	0.1707	0.3786	0	1
Railroad	0.1023	0.3032	0	1	0.0244	0.1552	0	1
Civil Engineering	0.3964	0.4895	0	1	0.5244	0.5025	0	1
Landscaping	0.0064	0.0798	0	1	0.0122	0.1104	0	1
Turn-key	0.6624	0.4732	0	1	0.7805	0.4165	0	1
Alternative	0.0639	0.2448	0	1	0.1220	0.3292	0	1
Technical Proposal	0.1023	0.3032	0	1	0.0732	0.2620	0	1
Lowest Price	0.1714	0.3771	0	1	0.0244	0.1552	0	1
Year 2007	0.0179	0.1327	0	1	0.0122	0.1104	0	1

Year 2008	0.1125	0.3162	0	1	0.1463	0.3556	0	1
Year 2009	0.2903	0.4542	0	1	0.1951	0.3987	0	1
Year 2010	0.1240	0.3298	0	1	0.1341	0.3429	0	1
Year 2011	0.1292	0.3356	0	1	0.1707	0.3786	0	1
Year 2012	0.0793	0.2704	0	1	0.1098	0.3145	0	1
Year 2013	0.0908	0.2875	0	1	0.1098	0.3145	0	1
Year 2014	0.0512	0.2204	0	1	0.0366	0.1889	0	1
Year 2015	0.0614	0.2402	0	1	0.0610	0.2408	0	1
Year 2016	0.0435	0.2041	0	1	0.0244	0.1552	0	1
Number of Observations	782				82			

5. Conclusion

This paper proposes a discriminant analysis in order to econometrically identify abnormally low bids (ALB) which are common in public construction procurement markets. The discriminant analysis utilizes a general mixture regression model. The unconditional probability of being ALB is separately specified as a logistic function.

We apply our discriminant analysis to Korean public construction procurement data between 2007 and 2016. As a result, 82 out of 864 bids are identified as ALB. After controlling the effect of ALB, we estimated the collusion effect in the market at about 1.09%p higher than the competitive price. The 1.09%p is almost one-fifth of 5.16%p, the collusion effect computed without considering ALB. It implies that researchers should be careful to identify ALB not to overestimate the collusion damage. We also find statistically significant effects of the number of bidders, construction types and bidding types on the price bid ratio of procurement contracts.

The estimation result of the logistic probability of ALB shows that the probability of ALB becomes higher as the number of bidders is bigger or the construction industry is in descending business cycle. Also, we find that architecture and railroad constructions significantly lower the probability of ALB than the other types of construction.

We have compared the 82 contracts which are discriminated as ALB to the other 782 'normal' contracts. The comparison shows us that the price bid ratio of ALB is only about two-third of the normal bids on average, and more than half of the ALBs in the data are civil engineering construction. We also find that turn-key type bidding tends to attract more ALB than lowest price type bidding.

Compliance with Ethical Standards:

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